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# Квантовая космология, SdS/CFT дуальность и происхождение инфляции

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# План

Волновая функция и матрица плотности черной дыры

Космология: волновая функция Хартла-Хокинга (no-boundary) и космологическая матрица плотности

Микроканоническая матрица плотности Вселенной и ее статсумма в модели конформной космологии

Космологические инстантоны как начальные данные для инфляционной Вселенной

Квантовое происхождение  $R^2$  модели Старобинского и неминимальной хиггсовской инфляции

Конформные поля высших спинов и решение проблем иерархии и стабильности квантовых эффектов (гравитационное обрезание)

Шварцшильд-де Ситтер/CFT duality: динамика тонких оболочек в ОТО

# Black hole wavefunction and density matrix

## Einstein-Rosen bridge

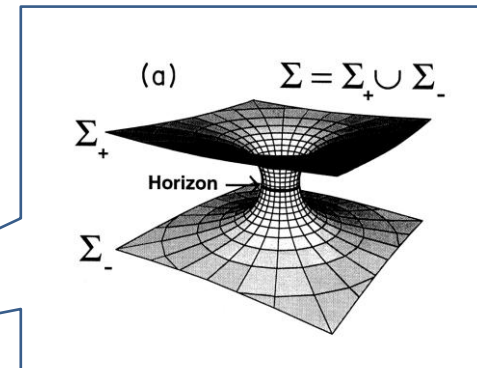
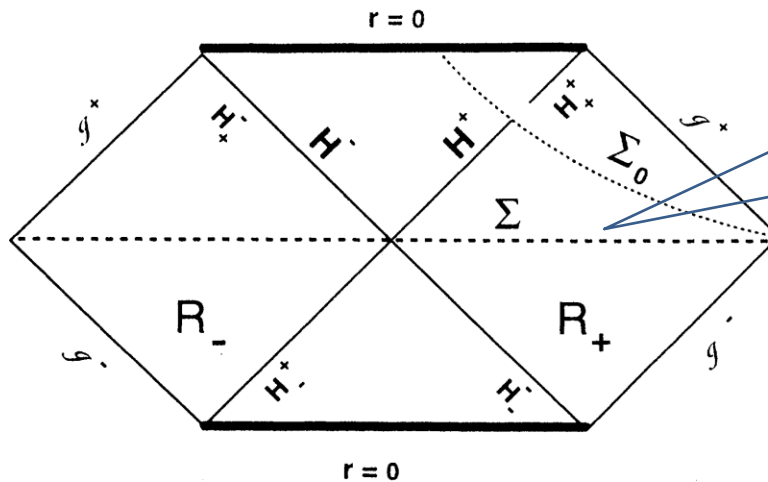
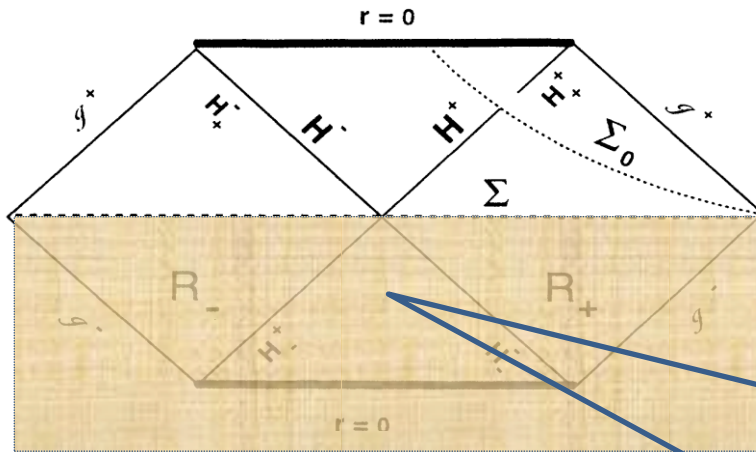


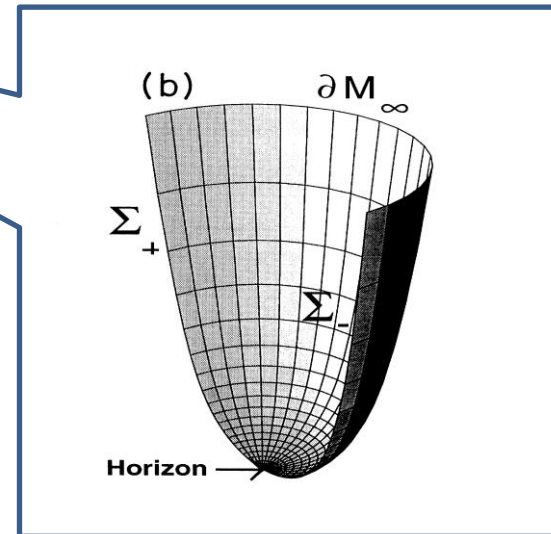
FIG. 2. This is a Penrose diagram of the eternal black hole. In Kruskal coordinates the global Cauchy surface  $\Sigma$  is defined by the equation  $U + V = 0$ . It has a wormhole topology  $\mathbf{R} \times \mathbf{S}^2$ . Both (future  $H^+$  and past  $H^-$ ) horizons consist of two parts  $H_{\pm}^+$  and  $H_{\pm}^-$ , the boundaries of  $R_{\pm}$ .



Half of BH "cigar" instanton:

Einstein-Rosen  
bridge

$$\Sigma = \Sigma_+ \cup \Sigma_-$$



No-boundary wavefunction of BH:

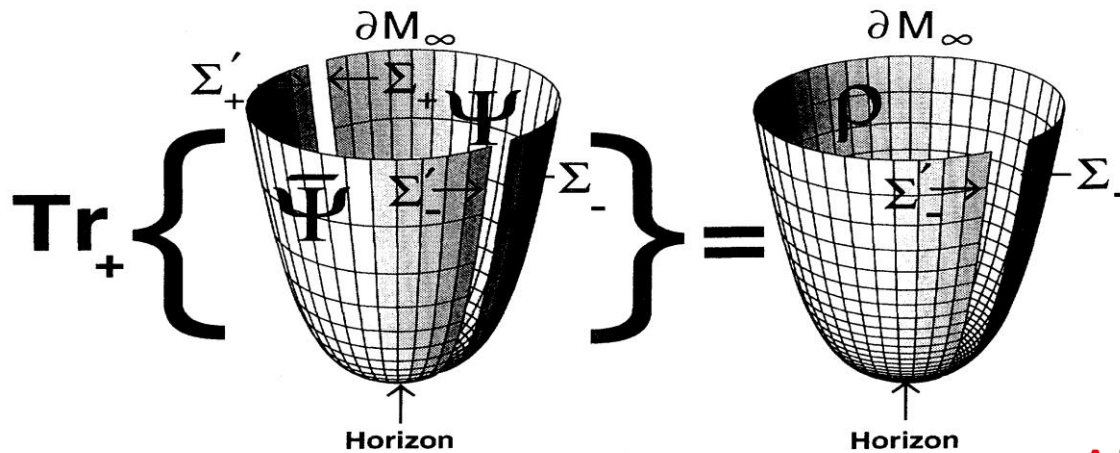
$$\Psi_{\Sigma}(\varphi_+, \varphi_-) = \int_{\phi_{\Sigma} = \varphi_{\pm}} d\phi e^{-S[\phi]}$$

A.B., V.P. Frolov and A.I. Zelnikov,  
Wavefunction of a Black Hole and  
The Dynamical Origin of Entropy,  
Phys. Rev. D51 (1995) 1741



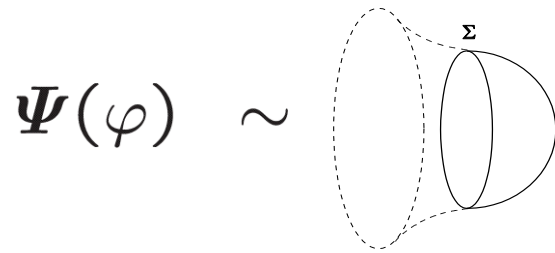
## Reduced density matrix of a black hole: tracing out and glueing

$$\rho(\varphi, \varphi') = \text{Tr}_+ | \Psi \rangle \langle \Psi | = \int D\varphi_+ \Psi(\varphi_+, \varphi) \Psi^*(\varphi_+, \varphi')$$

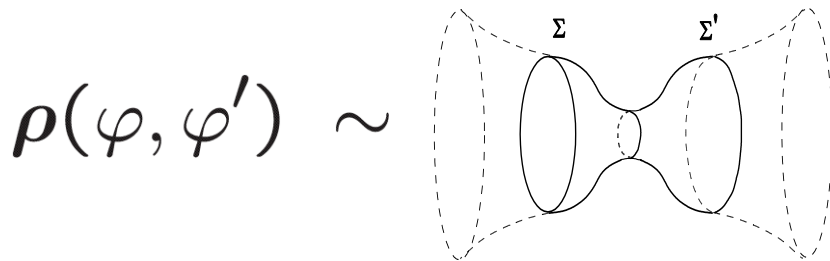


A.B.,V.P.Frolov and A.I.Zelnikov,  
Wavefunction of a Black Hole and  
The Dynamical Origin of Entropy,  
Phys. Rev. D51 (1995) 1741

# Cosmology: no-boundary wavefunction and cosmological density matrix



$$\Psi(\varphi) = \int_{\phi|_{\Sigma}=\varphi} d\phi e^{-S[\phi]}$$



$$\rho(\varphi, \varphi') = \int d\phi e^{-S[\phi]} \Big|_{\phi|_{\Sigma}=\varphi, \phi|_{\Sigma'}=\varphi'}$$

# Cosmological initial conditions – microcanonical density matrix of the Universe and its statistical sum

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

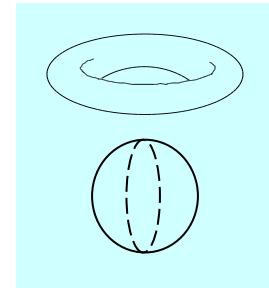


A.B., Phys. Rev. Lett.  
99, 071301 (2007)

Statistical sum

$$e^{-\Gamma} = \int_{\text{periodic}} D[g_{\mu\nu}, \Phi] e^{-S[g_{\mu\nu}, \Phi]}$$

on  $S^3 \times S^1$  (thermal)  
including as a limiting  
(vacuum) case  $S^4$



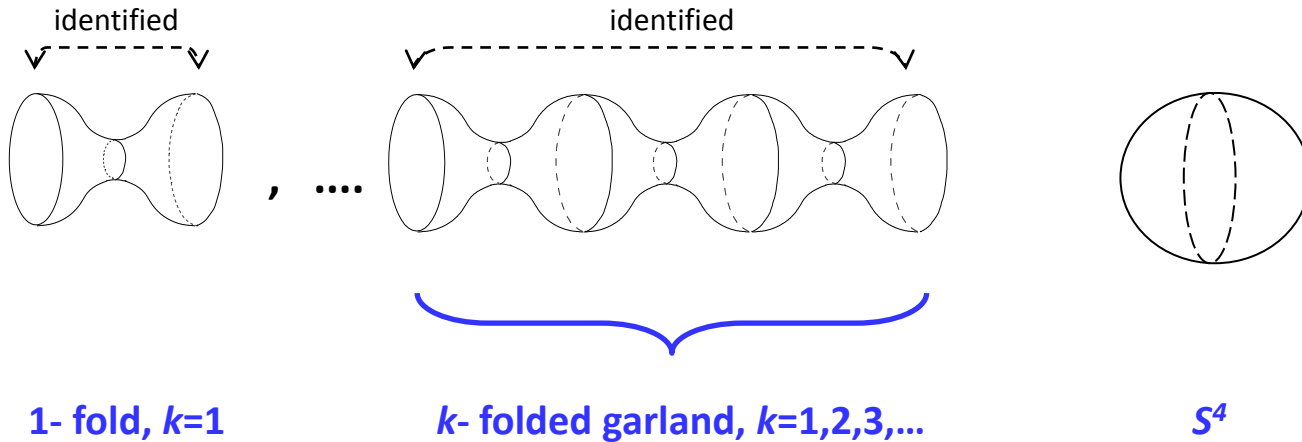
Application to CFT driven cosmology -- Universe dominated by quantum matter conformally coupled to gravity (CFT) :

$$S[g_{\mu\nu}, \Phi] = -\frac{M_P^2}{2} \int d^4x g^{1/2} (R - 2\Lambda) + S_{CFT}[g_{\mu\nu}, \Phi]$$

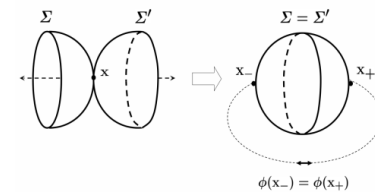
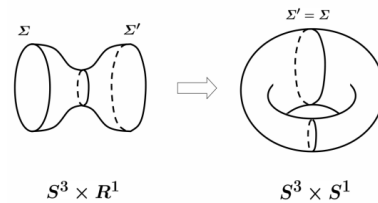
$\Lambda$  -- primordial  
cosmological constant



Saddle point solutions for  $S_{eff} [ g_{\mu\nu} ]$  with Friedmann-Robertson-Walker geometry --- set of periodic – **thermal** -- garland-type instantons with oscillating scale factor (  $S^1 \times S^3$  ) and the vacuum Hartle-Hawking instantons (  $S^4$  )



Examples of identification



# Properties and implications of cosmological instantons

## 1) Limited range of $\Lambda$ – subplanckian domain (limiting the string vacua landscape?):

$$\Lambda_{\min} \leq \Lambda \leq \Lambda_{\max} = \frac{12\pi^2 M_P^2}{\beta}$$

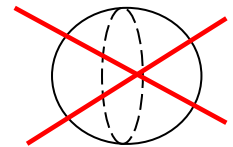
$$\beta = \sum_s \beta_s N_s$$

$N_s$  # of fields of spin  $s$

Conformal anomaly coefficient

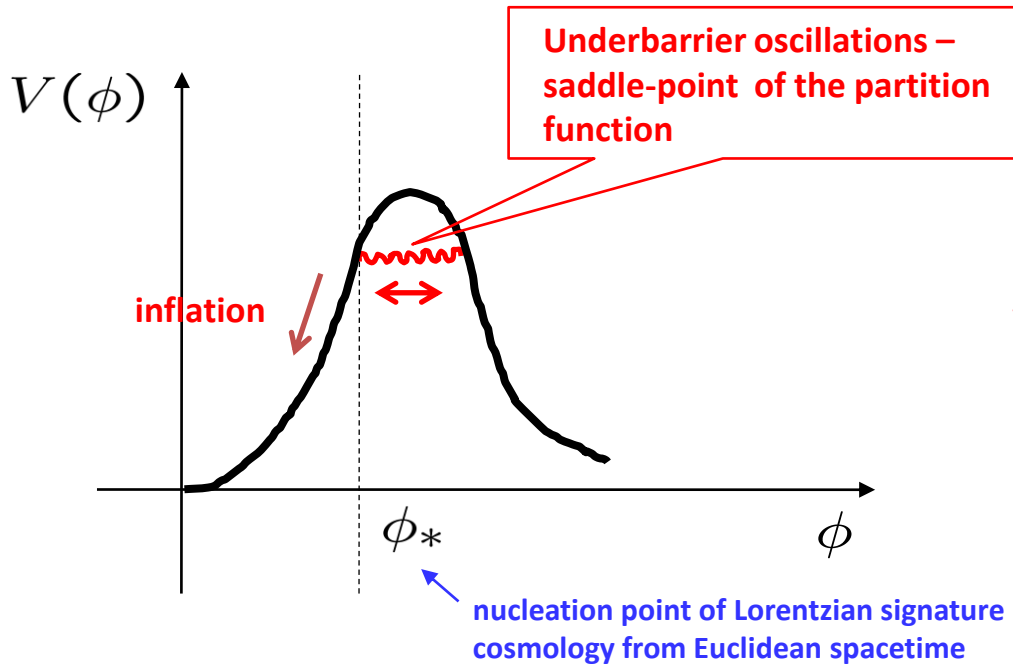
$$g_{\mu\nu} \frac{\delta \Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{64\pi^2} g^{1/2} (\beta E + \alpha \square R + \gamma C_{\mu\nu\alpha\beta}^2)$$

## 2) No-boundary instantons $S^4$ are ruled out by *infinite positive* Euclidean action – elimination of infrared catastrophe

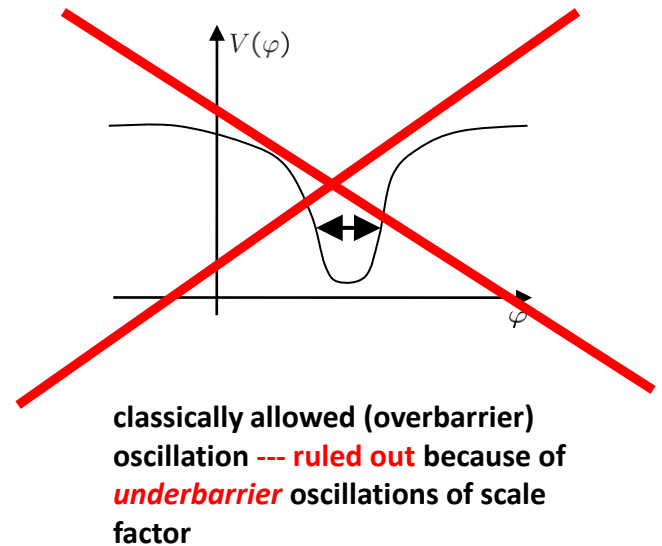


## 3) Generalization to inflationary model, $\Lambda \rightarrow V(\phi)$ – selection of inflaton potential $V(\phi)$ maxima (new type of hill-top inflation) – quantum origin of the Starobinsky model and Higgs inflation model with a *primordial radiation* at $V(\phi) \sim \Lambda_{\max}$

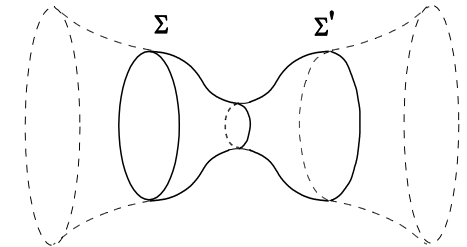
# Selection of inflaton potential *maxima* as initial conditions for inflation – new paradigm of **HILL-TOP INFLATION**



VS



Lorentzian Universe with initial conditions set by the instanton --  
analytic continuation of the instanton solutions.  
Expansion and quick dilution of primordial radiation decay  
of a composite  $\Lambda$ , exit from inflation and particle creation  
of conformally **non-invariant** matter and its thermalization





#### 4) Origin of Starobinsky $R^2$ model and non-minimal Higgs inflation: slow roll smallness parameters + thermal imprint on primordial power spectrum

$$\eta = -\frac{\beta\mu^2}{12\pi^2 M_P^2} < 0$$

$$\epsilon = \frac{1}{2} \left( \frac{\Delta\phi}{M_P} \right)^2 \eta^2 \sim \eta_*^2 \ll |\eta|$$



$$n_s = 1 - 6\epsilon + 2\eta \simeq 0.96$$

$$r = 16\epsilon \ll 1$$

Main observable CMB parameters (WMAP, Planck,...):

$$\frac{\Delta T}{T} \sim 10^{-5}, \quad n_s \simeq 0.96, \quad r \simeq 0.003$$

relation of CMB data to the Higgs mass

A.Kamenshchik, A.Starobinsky & A.B. (2008)

Bezrukov,Shaposhnikov (2008);

$$M_{\text{Higgs}} \simeq 126 \text{ GeV}$$

#### 5) Hidden sector of conformal higher spin fields (CHS): solution of hierarchy problem and stabilization of graviton loop corrections below the gravitational cutoff

$$\Lambda \sim 10^{-11} M_P^4 \ll \Lambda_{\text{cutoff}}$$

A.Barvinsky (2016)

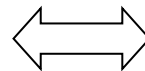
# Schwarzschild-de Sitter/CFT duality

The analogue of the **thermal** version of AdS/CFT correspondence:  
 duality of the 4D finite temperature boundary CFT to 5D black hole  
 thermodynamics in AdS spacetime with a boundary

Witten (1998)

**VS**

4D CFT cosmology: Einstein theory sourced  
 by quantum conformal matter with  
 conformal anomaly at finite temperature



Brane induced gravity in 5D Schwarzschild-  
 deSitter bulk (5D black hole in  $dS_5$ )

$$G_4, \Lambda_4, \mathcal{C}$$

primordial 4D  
 cosmological constant

4D radiation is imitated by the  
 BH mass

$$\mathcal{C} \sim G_5 M = R_S^2$$

$$G_5, \Lambda_5, R_S$$

Schwarzschild  
 radius of bulk BH

Technique of thin shells in GR:  
 Einstein equations in the bulk

+

Israel junction conditions on the brane

V.A.Berezin, V.A.Kuzmin and  
 I.I.Tkachev, Thin Wall Vacuum  
 Domains Evolution,  
 Phys.Lett. B120 (1983) 91

# Duality of 4D CFT driven cosmology and 5D brane induced gravity

4D side

$$S_E[g_{\mu\nu}, \phi] = -\frac{1}{16\pi G} \int d^4x g^{1/2} (R - 2\Lambda) + S_{CFT}[g_{\mu\nu}, \phi]$$

$\Lambda$  -- primordial cosmological constant

$N_s \gg 1$  conformal fields of spin  $s=0,1,1/2$

5D side

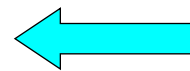
$$S[G_{AB}(X)] = -\frac{1}{16\pi G_5} \int_{\text{Bulk}} d^5X G^{1/2} (R^{(5)}(G_{AB}) - 2\Lambda_5) - \int_{\text{brane}} d^4x g^{1/2} \left( \frac{1}{8\pi G_5} [K] + \frac{1}{16\pi G_4} R(g_{\mu\nu}) \right).$$

5D Schwarzschild-dS solution with a bulk black hole of the mass  $\sim R_s^2/G_5$  and a spherical 4D shell inhabited by the Einstein term

$$ds_{(5)}^2 = f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega_{(3)}^2$$

$$f(R) = 1 - \frac{\Lambda_5}{6} R^2 - \frac{R_S^2}{R^2}$$

embedding



$$ds_{(4)}^2 = d\tau^2 + a^2(\tau) d\Omega_{(3)}^2$$

$$R = a(\tau), \quad T = T(\tau), \quad T'(\tau) = \frac{\sqrt{f(a) - a'^2}}{f(a)}$$

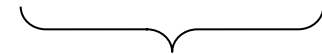


# Euclidean Schwarzschild-de Sitter “cigar” instanton:

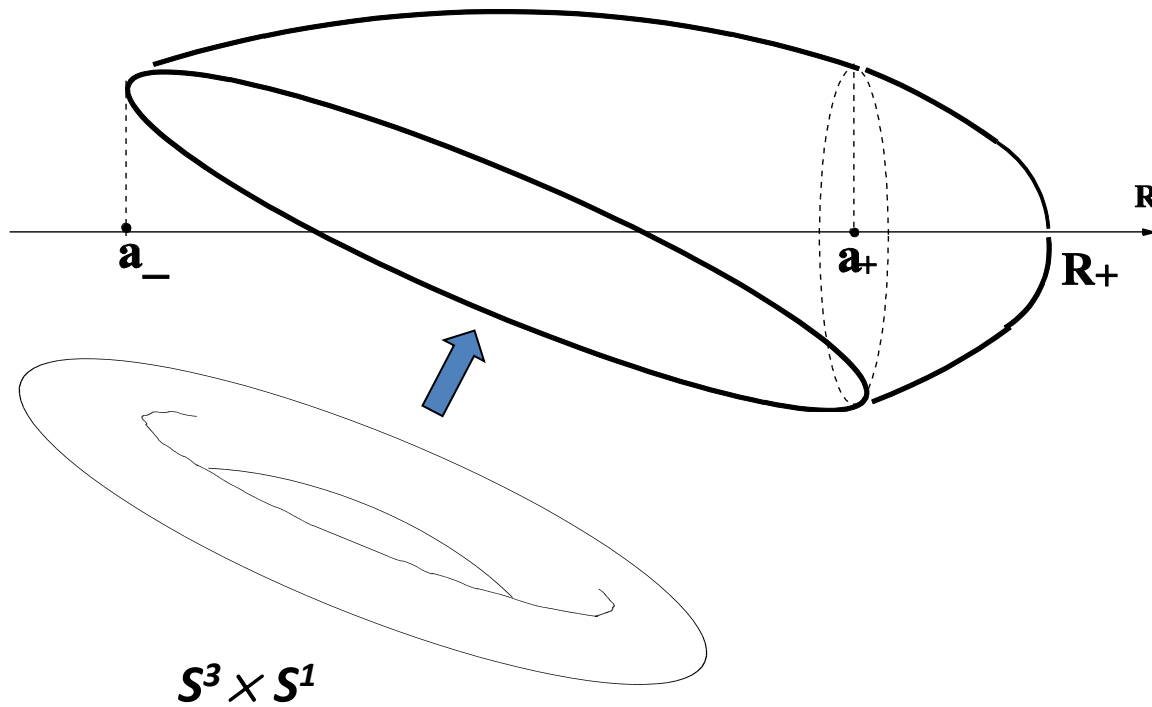
$$f(R) \geq 0, \quad R_- \leq R \leq R_+$$

$$R_{\pm}^2 = \frac{3}{\Lambda_5} \left( 1 \pm \sqrt{1 - 2\Lambda_5 R_S^2/3} \right)$$

$$R_- < a_- \leq a(\tau) \leq a_+ < R_+$$



4D instanton domain





**Желаем нашим лауреатам  
дальнейших успехов, творческих  
прозрений и талантливых учеников !**